23. The fish dragged his boat x km east, then $6x + 3$ km north, then $x + 8$ km east, then $2x - 2$ km south, then $2x + 8$ km west, then x^2 km south. The distance to the north must equal the distance to the south, so $6x+3 = 2x-2 + x^2$. Thus, $x^2 - 4x - 5 = 0 = (x - 5)(x + 1)$. The only possible value of x is 5, so substitute it in for all distances: $5 + 6(5)+3 + 5+8 + 2(5)-2 + 2(5)+8 + 5^2 = 102$.

A) 5 B) 30 C) 81 D) 102

23. D

24. I mixed 400 ml of 30% sugar lemonade and 200 ml of 40% sugar lemonade. That gave me 120 ml + 80 ml = 200 ml of sugar out of 600 ml of lemonade in the two batches. During the week 100 ml of pure water evaporated, leaving 200 ml sugar in 500 ml lemonade.

A) 30% B) 33% C) 35% D) 40%

24. D

25. If $\sqrt{xy} \times \sqrt{15} = \sqrt{3x^2} \times \sqrt{y}$, then $15xy = 3x^2y$, and $x = 5$.

A) 5 B) y C) $5y$ D) $5 + y$

25. A

26. $\dfrac{x^2-3x-18-x+6}{(x-6)(x+3)} = \dfrac{x^2-4x-12}{(x-6)(x-3)} = \dfrac{(x-6)(x+2)}{(x-6)(x-3)} = \dfrac{x+2}{x+3}$.

A) 1 B) $\dfrac{x+2}{x+3}$ C) $\dfrac{x-6}{x+3}$ D) $\dfrac{-2}{-(x-6)}$

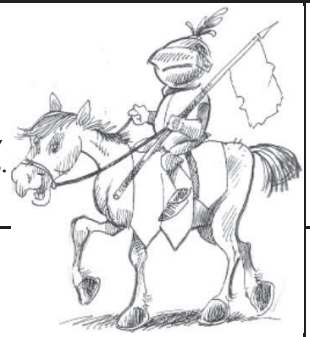
26. B

27. $3(3x - 4y + 5z = 13) - 2(4x - 5y + 6z = 18) \Longrightarrow x - 2y + 3z = 3$.

A) 1 B) 3 C) 15 D) 28

27. B

28. Between 1000 and 5000 knights were at the start. Each day 2/3 of the remaining knights fell or fled. Yesterday Saul lost his final 2 fellow knights. Work backward: Yesterday there were 3, the day before 9, before that 27, then 81, then 243, then 729, then 2187. 7 days.



A) 70 B) 14 C) 7 D) 6

28. C

29. If $8^{2a} = 32b$, $(2^3)^{2a} = 2^5b$, $(2^{6a})/2^5 = b = 2^{6a-5}$.

A) 2^a B) $2^{6a/5}$ C) 2^{2a-3} D) 2^{6a-5}

29. D

30. Find factors of 2: 1000/2=500, 1000/4=250, 1000/8=125, 1000/16=62…. There are 500+250+125+62+31+15+7+3+1=994 2's. $2^{994}=(2^2)^{497}=4^{497}$.

A) 250 B) 312 C) 330 D) 497

30. D

The end of the contest **A**

MATH

ALGEBRA COURSE 1 CONTEST

Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

Information & Solutions

A

Spring, 2020

Directions for Grading

- **Date** You may give this contest any time after April 15. The *Algebra Course 1 Contest* is for use in your own school or district. We've enclosed a registration form for next year. Instructions for optionally submitting results are included on a separate sheet entitled "*Using the Score Report Center*."
- **Urgent questions?** Write to comments@mathleague.com, or call 1-201-568-6328 or 1-516-365-5656.
- **Scores** Remind students that *this is a contest, and not a test*—there is no "passing" or "failing" score. Few students score as high as 24 points (80% correct); students with half that, 12 points, *should be commended!*
- **Solutions** Detailed solutions appear in each question box, and letter answers are in the *Answers* columns on the right. You may copy this solution key and give a copy to every student who took this contest.
- **Awards** The original contest package contained 1 book award (and a bookplate you should affix to the book's inside front cover) for the 1st place student. We also enclosed 5 *Certificates of Merit*—1 each for the runner-up on each grade level, plus extras for ties.
- **Additional Book Awards & Additional Certificates** If you want to give more than 1 book award, you may purchase additional books as described below. Do you need more Certificates of Merit? If so, send your name, school, and school mailing address to our mailer at: ***Math Certificates, P.O. Box 17, Tenafly, NJ 07670-0017.*** *Include a self-addressed, stamped envelope (**2 stamps required**) large enough to hold certificates.*

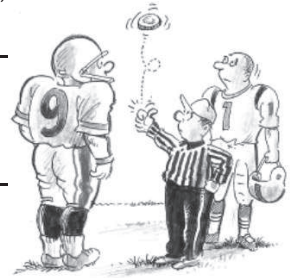
  The school's top scorer will receive the book *Math Contests—High School (Vol. 5).* Other high scorers will receive Certificates of Merit. In any one school year, no student may win both a book and a certificate. The book and certificates were in the original contest package.

If needed, duplicate book awards may be ordered as described below.

Answers

1. If $T = 1$, $O = 2$, and $1+2+S+S = 7$, then $2S = 4$, and $S = 2$. **1. A**

A) 2 B) 3 C) 3.5 D) 4



2. If x is an integer, then the least possible value of $4x^2$ is obtained when $x = 0$. **2. B**

A) -4 B) 0 C) 4 D) 16

3. $(c^{20})(c^2)(c^0) = c^{20+2+0} = c^{22}$. **3. C**

A) 0 B) c^0 C) c^{22} D) c^{40}

4. I had g invited guests. Each invited guest brought 2 uninvited friends, for $2g$ additional people. Each person brought two gifts. Multiply total people ($g + 2g$) by the number of gifts each brought, 2. **4. C**

A) $(g + 2) \times 2$ B) $(g \times 2) + 2$ C) $(g + 2g) \times 2$ D) $(g + 2g) \times 2g$

5. $4y(x-y) - (3x+2y)(x-y) = [4y - (3x+2y)](x - y) = [4y-3x-2y](x - y)$. **5. D**

A) $(6y + 3x)(x - y)$ B) $(6y - 3x)(x - y)$
C) $(2y + 3x)(x - y)$ D) $(2y - 3x)(x - y)$

6. $4x^2 + 3x + 2x^3 - 2x^2 - 3x - 4x^3 = (3x - 3x) + (4x^2 - 2x^2) + (2x^3 - 4x^3)$. **6. B**

A) 0 B) $2x^2 - 2x^3$ C) $2x^2 + 6x - 2x^3$ D) $2x^2 + 6x + 6x^3$

7. If $\frac{3}{5}(2y) = \frac{4}{7}x$, multiply by 5/3: $2y = \frac{20}{21}x$. Divide by 2 to get $\frac{10}{21}x$. **7. A**

A) $\frac{10}{21}x$ B) $\frac{20}{21}x$ C) $\frac{21}{20}x$ D) $\frac{21}{10}x$

8. $(x + 2)(x - 2)(x^2 - 4) = 0 = (x + 2)^2(x - 2)^2$, so x can be -2 or 2. **8. B**

A) 1 B) 2 C) 3 D) 4

9. If $x > 5$ and prime, the l.c.m. of $2^2 5^1 x^2$ and $2^1 3^1 5^1 x^3$ is $2^2 3^1 5^1 x^3$. **9. B**

A) $10x$ B) $60x^3$ C) $60x^5$ D) $600x^5$

10. Water time:walking time is 30 sec.:120 sec. = 1:4, so they spend 1/5 of the h hrs. in the water. Since h hrs. = $60h$ minutes, they will spend $60h/5$ min. = $12h$ min. in the water. **10. A**

A) $12h$ B) $24h$ C) $36h$ D) $48h$

11. If $x = y+1$, then $(y+1)^2 - y^2 = 2y+1 = 39$; $y = 19$, $x = 20$. **11. C**

A) 39 B) 78 C) 380 D) 1521

12. $x^3 - x^2 + x - 1 = x^2(x-1) + (x-1) = (x^2+1)(x-1)$ is divisible by $x-1$, so R = 0. **12. A**

A) 0 B) 1 C) x D) $2x$

Answers

13. A $\perp$ line has negative reciprocal slope, so $y = -3x + b$. Substitute 0 for y in original line to find x-intercept -12; use to find new b. **13. D**

A) $y = -3x + 4$ B) $y = 3x - 36$ C) $y = \frac{1}{3}x + 4$ D) $y = -3x - 36$

14. The sum of all solutions is $-b/a$, so for $4x^2 - 4x - 35 = 0$: $-(-4)/4 = 1$. **14. C**

A) -1 B) 0 C) 1 D) 4

15. If $b^2 + 31 = g^4 = (g^2)^2 = (b + 1)^2$, then $b^2 + 31 = b^2 + 2b + 1$, so $b = 15$. Substitute $b = 15$ in $b + 1 = g^2$; $16 = g^2$ and $g = 4$. **15. A**

A) 4 B) 11 C) 12 D) 30

16. Rate r m/min.$= 60r$ m/hr.$= 60r/1000$ km/hr. Divide distance k by rate to find time. **16. C**

A) $\frac{1000}{rk}$ B) $\frac{60k}{1000r}$ C) $\frac{1000k}{60r}$ D) $\frac{1000}{60rk}$

17. $4^{2x} + 4^{2x} + 4^{2x} + 4^{2x} = 4(4^{2x}) = 4^{2x+1} = (2^2)^{2x+1} = 2^{4x+2}$. **17. B**

A) 2^{4x} B) 2^{4x+2} C) 4^{8x} D) 16^{2x}

18. If $f(x) = 8x^2 - 2$, then $f(4) = 8(4)^2 - 2 = 126$. $f(-4)$ will also yield 126. **18. D**

A) $f(126)$ B) $f(8)$ C) $f(-2)$ D) $f(-4)$

19. If $3x - 7 < 5$, $x < 4$. If $3x - 7 > -5$, $x > 2/3$. So $2/3 < x < 4$. 1, 2, and 3 fit. **19. C**

A) 1 B) 2 C) 3 D) 6

20. Multiply $x^2+x+1 = 18$ by x to get $x^3+x^2+x = 18x$. The average is $18x/3$. **20. A**

A) $6x$ B) $9x$ C) $18x$ D) $36x$

21. The Cones is an elite *a cappella* vocal group. Together, 4 Cones working at the same rate can set up every chair in the theater in 56 min. So it takes $4 \times 56 = 224$ Cone-min. of work to set up every chair. Divide 224 Cone-min. by 7 Cones to find that it will take 32 min. for 7 Cones working at the same rate to set up every chair in the theater. **21. B**

A) 24 B) 32 C) 48 D) 98

22. $10^a = (0.01/100) \times 10^b = (10^{-2}/10^2) \times 10^b = 10^{-4} \times 10^b = 10^{-4+b}$, so $a = b - 4$. **22. A**

A) $b - 4$ B) $b - 2$ C) $b + 2$ D) $b + 4$