# Problems of the Week

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## Week 1

1. (2000 AMC12) Two different primes between 4 and 18 are chosen When their sum is subtracted from their product, which of the following numbers could be obtained?

   (A) 22 (B) 60 (C) 119 (D) 194 (E) 231

2. How many consecutive zeros are at the end of $2020! = 1 \times 2 \times 3 \times \cdots \times 2020$?

3. Refer to Figure 1. $H$ is an arbitrary point in rectangle $ABCD$. We denote the area of $\Delta XYZ$ by $[XYZ]$. If $[HBC] = 12$, $[AHB] = 5$, then $[HBD] = ?$

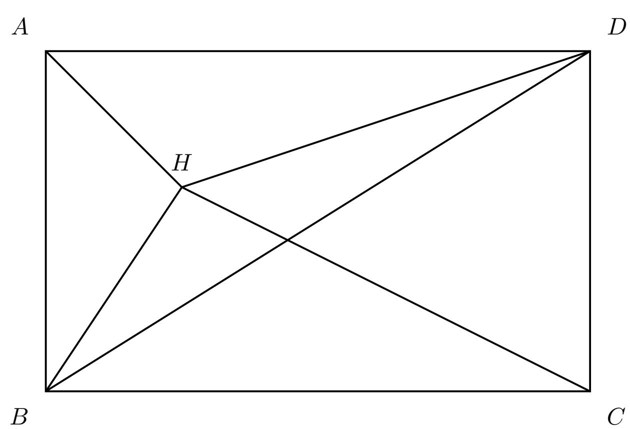


Figure 1

## Week 1 Answers

1. (From AoPS) (Solution 1) Any two prime numbers between 4 and 18 have an odd product and an even sum. Any odd number minus an even number is an odd number, so we can eliminate B, D, and A. Since the highest two prime numbers we can pick are 13 and 17, the highest number we can make is $(13)(17) - (13 + 17) = 221 - 30 = 191$. Thus, we can eliminate E. So, the answer must be $\boxed{\textbf{(C)}\ 119}$.

   (Solution 2) Let the two primes be $p$ and $q$. We wish to obtain the value of $pq - (p + q)$, or $pq - p - q$. Using Simon's Favorite Factoring Trick, we can rewrite this expression as $(1 - p)(1 - q) - 1$ or $(p - 1)(q - 1) - 1$. Noticing that $(13 - 1)(11 - 1) - 1 = 120 - 1 = 119$, we see that the answer is $\boxed{\textbf{(C)}\ 119}$.

   (Solution 3) The answer must be in the form $pq - p - q = (p - 1)(q - 1) - 1$. Since $p - 1$ and $q - 1$ are both even, $(p - 1)(q - 1) - 1$ is $3 \pmod 4$, and the only answer that is $3 \pmod 4$ is $\boxed{\textbf{(C)}\ 119}$.

2. The number of consecutive ending 0's is determined by the number of 2 and 5 factors in 2020!. Since the number has way more factor 2's than factor 5's, it suffices to count the number of factor 5's.

   All numbers less than or equal to 2020 that's divisible by 5 contributes a factor 5 to 2020!; there are $2020 \div 5 = 404$ of them. Numbers divisible by 25 contribute an extra factor of 5; there are $\lfloor 2020 \div 25 \rfloor = 80$ of them ($\lfloor x \rfloor$ means rounding $x$ down to the nearest integer). Numbers divisible by 125 contribute an extra; there are $\lfloor 2020 \div 125 \rfloor = 16$ of them. Numbers divisible by 625 contribute an extra; there are 3 of them.

   In total, 2020! has
   $$404 + 80 + 16 + 3 = 503$$
   factor 5's and therefore $\boxed{503}$ consecutive ending 0's.

3. The key observation is that $[AHB] + [DHC] = \frac{1}{2}[ABCD]$. One can see that by drawing a vertical line through $H$ which divides $ABCD$ into two adjacent rectangles, and each of $\Delta AHB$ and $\Delta DHC$ are half the area of one of the rectangles.

   Therefore we can set up the following equation
   $$[AHB] + [DHC] = \frac{1}{2}[ABCD] = [HBC] + [DHC] - [HBD]$$
   $$\Rightarrow [AHB] = [HBC] - [HBD]$$
   $$\Rightarrow [HBD] = [HBC] - [AHB] = 12 - 5 = \boxed{7}.$$